Quantum mechanics. Department of physics. 7th semester.

Lesson № 17. ***Spin (ending)***

1. Checking hometask.

1.1. Find eigenvalues and eigenfunctions of the operator $\vec{a}\cdot\hat{\vec{S}}$, where $\vec{a}$ is a 3D vector, $\hat{\vec{S}}$ is the spin-½.operator.

1.2. Calculate $\overline{\hat{S}_n^2}$ for the operator of a spin projection $\hat{S}_n$ on an arbitrary direction, which is defined by the unit vector $\vec{n}=(n_x,n_y,n_z)=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)$. (from НКК № 5.2)

2. Tasks solving.

Task 1. For the spin $S=1/2$ indicate raising and lowering (ladder) operators $S_\pm = S_x \pm iS_y$, find their action on eigenfunctions of the operator S_z, find $\left(S_\pm\right)^2$ and $S_+S_- + S_-S_+$ anticommutators. (НКК № 5.11)

Task 2. For two particles with spin ½ find eigenfunctions of the operator $\hat{\vec{S}}=\hat{\vec{S}}_1+\hat{\vec{S}}_2$, namely common eigenfunctions of the operators $\hat{\vec{S}}^2$ and S_z (НКК № 5.17 or Flugge, Part 2, task № 139)

As reference. Adding momentum rule is $S=S_1+S_2, S_1+S_2-1,...,|S_1-S_2|$

Spin functions of two particles with spin $S_1=S_2=1/2$

Eigenfunctions	Eigenvalues			Symmetry
	S	$\lambda_{S^2}=S(S+1);$	S_z	
$\left\|\uparrow\right\rangle_1\cdot\left\|\uparrow\right\rangle_2$	1	2	$S=+1$	**Triplet** $S=1$ (symmetrical spin wave function)
$\frac{1}{\sqrt{2}}\left(\left\|\uparrow\right\rangle_1\cdot\left\|\downarrow\right\rangle_2+\left\|\downarrow\right\rangle_1\cdot\left\|\uparrow\right\rangle_2\right)$	1	2	$S=0$	
$\left\|\downarrow\right\rangle_1\cdot\left\|\downarrow\right\rangle_2$	1	2	$S=-1$	
$\frac{1}{\sqrt{2}}\left(\left\|\uparrow\right\rangle_1\cdot\left\|\downarrow\right\rangle_2-\left\|\downarrow\right\rangle_1\cdot\left\|\uparrow\right\rangle_2\right)$	0	0	0	**Singlet** $S=0$ (antisymmetric spin wave function)

Here $\left|\uparrow\right\rangle_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\left|\downarrow\right\rangle_{1,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ eigenfunctions of the operator $S^z_{1,2}$, corresponding to eigenvalues +1/2 and –1/2 agreeably , for the first and second particles.

<u>Hometask</u> НКК 5.17 (finish)

LL – Landau LD, Lifshits IM, Quantum Mechanics
НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Additional: Flugge Z. Problems in quantum mechanics. P.1, P.2.1974